Erratum to
“The Ricean Objection: An Analogue of Rice’s Theorem for First-Order Theories”

IGOR CARBONI OLIVEIRA and WALTER CARNIELLI, Centre for Logic, Epistemology and the History of Science (CLE) and Department of Philosophy (IFCH), State University of Campinas - UNICAMP, C.P. 6133 - 13083-970 Campinas-SP, Brazil.

There is an error in the proof of the main theorem on page 6 (“Rice’s Theorem for Logic”). If $\phi$ is undecidable in $Q$, the claim that $Ax1$ and $Ax2$ are tautologies is not valid, and this fact compromises the rest of the proof. Furthermore, the following decidable property is a counter-example to our result:

Definition 1. Let $T$ be a theory. A theory $T^+$ is an extension of $T$ if $Th(T) \subseteq Th(T^+)$.

Definition 2. $Sub(T)$ denotes the set $\{Th(T^-) : T$ is an extension of $T^-\}$.

Proposition 1. If $T$ is a consistent decidable theory, then $Sub(T)$ is a non-trivial decidable property.

Proof. By the consistency of $T$, $Sub(T)$ is non-trivial. For each finite $A \subseteq L_S$, the following holds:

$$Th(A) \in Sub(T) \iff T$ is an extension of $A \iff T \vdash \bigwedge_{\phi \in A} \phi.$$ 

Since $T$ is decidable, there exists an algorithm which decides for given finite $A \subseteq L_S$ whether $T \vdash \bigwedge_{\phi \in A} \phi$ or not. That is, $Sub(T)$ is decidable.

For instance, $Sub(\forall x \forall y (x = y))$ is a counter-example to our result. This negative result, which is contrary to our initial intuition, led us to consider the existence of these “Ricean” undecidability results in a more general sense.
Erratum

Definition 3. Let $T$ be a theory and $\Gamma$ be a set of sentences. $\Gamma$ is a property on $T$ if the following holds for any sentences $\phi$ and $\psi$:

$$T \vdash \phi \leftrightarrow \psi \Rightarrow [\phi \in \Gamma \iff \psi \in \Gamma].$$

A property $\Gamma$ is trivial if it is the empty set or the set of all sentences.

In particular, note that a property $P$ in the sense of our paper corresponds to the property $\{\bigwedge_{\phi \in A} \Phi: Th(A) \in P$ and $A$ is finite\} on $\emptyset$. If we consider sufficiently expressive theories such as $Q$ (Robinson arithmetic), it is indeed possible to prove that they are undecidable in the sense of Rice’s theorem.

Theorem 1. Every non-trivial property on $Q$ is undecidable.

For instance, since $Th(Q)$ is a property on $Q$, we derive as a particular case of this general result that $Q$ is an undecidable theory. Theorem 1 is a consequence of the diagonal lemma, and the reader is referred to [1] for a general treatment of this elegant result and its consequences. A new interpretation of this result will appear in a forthcoming paper.

Acknowledgements

The flaw in our proof was pointed out to us by professor Stewart Shapiro (Department of Philosophy, Ohio State University). Hirofumi Yoshikawa (Tokyo Institute of Technology) provided the counter-example and brought [1] to our attention. We would also like to thank him for useful discussions on the subject.

References